COMPLEX NUMBER

EXERCISE – I

HINTS & SOLUTIONS

Sol.1 **A**

$|z| = 4$; $\arg(z) = 4 \cdot e^{i\frac{5\pi}{6}}$

$z = |z| \arg(z) = 4 \cdot e^{i\frac{5\pi}{6}}$

$= 4\left[\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right]$

$= 4\left[\frac{-\sqrt{3}}{2} + i\sin\frac{1}{2}\right] = -2\sqrt{3} + 2i$

Sol.2 **C**

$z = \sin\frac{6\pi}{5} + i\left(1 + \cos\frac{6\pi}{5}\right)$

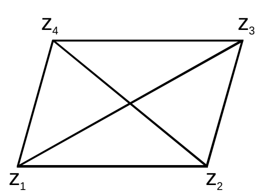
$= -\sin\left(\frac{\pi}{5}\right) + i\left[1 - \cos\frac{\pi}{5}\right]$

$= -\sin\frac{\pi}{5} + i\left[1 - \left(1 - 2\sin^2\frac{\pi}{10}\right)\right]$

$\arg(z) = \tan\theta = \dfrac{2\sin^2\frac{\pi}{10}}{-2\sin\frac{\pi}{10}\cos\frac{\pi}{10}} = -\tan\frac{\pi}{10}$

$\tan\theta = \tan\left(\pi - \frac{\pi}{10}\right)$

Sol.3 **B**



$\frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2}$

$z_1 + z_3 = z_2 + z_4$

Sol.4 **B**

Let $z = x + iy$ $\quad z^2 = x^2 - y^2 - y^2 + 2ixy$

$Re(z)^2 = x^2 - y^2$

A rectangular Hyperbola

Sol.5 **D**

$|z - 4| < |z - 2|$ Let $z = x + iy$

$|(x - 4) + iy| < |(x - 2) + iy|$

$(x - y)^2 + y^2 < (x - 2)^2 + y^2$

$x^2 - 8x + 16 + y^2 - 4x + 4 + y^2$

$4x > 12$

$x > 3$

$Re(z) > 3$

Sol.6 **A**

Let $\quad z = x + iy \quad |z| = 2$

$Re(z^2) = 0 \quad x^2 + y^2 = 4$

$\Rightarrow \quad x^2 - y^2 = 0$

$2x^2 = 4 \Rightarrow x^2 = 2 \quad \Rightarrow x = \pm\sqrt{2}$

$(\sqrt{2}, \sqrt{2}), (\sqrt{2}, -\sqrt{2}), (-\sqrt{2}, \sqrt{2})$

Sol.7 **A**

$\frac{z-1}{z+1} = -\left(\frac{\bar{z}-1}{\bar{z}+1}\right)$

$(z - 1)(\bar{z} + 1) = -(z + 1)(\bar{z} - 1)$

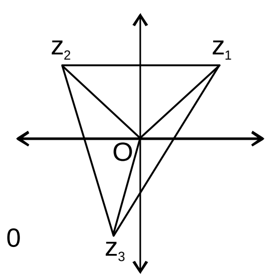
$z\bar{z} + z - \bar{z} - 1 = -z\bar{z} + z - \bar{z} + 1$

$z\bar{z} = 1 \quad \Rightarrow |z| = 1$

Sol.8 **A**

$|z_1| = |z_2| = |z_2| = 1$

origin will be the circumcentre of triangle and it is a equilateral O so centroid & circumcentre will be same



So $\frac{z_1 + z_2 + z_3}{3} = 0$

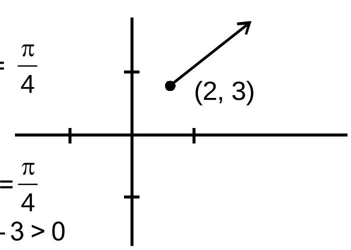
$z_1 + z_2 + z_3 = 0$

Sol.9 **A**

Arg $(z - z - 3i) = \frac{\pi}{4}$

Let $z = x + iy$

Arg $((x - 2) + i(y - 3)) = \frac{\pi}{4}$



and $x - x > 0$; $y - 3 > 0$

$\tan^{-1}\frac{y-3}{x-2} = \frac{\pi}{4} \qquad x > 2$; $y > 3$

$y - 3 = x - 2$

$y = x + 1$

Sol.10 **A**

By figure we can say that Arg $= \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and it is inner area of circle.

Sol.11 C

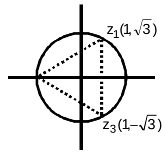
$|z| = 2$

$\Rightarrow$ a circle withs radius = 2

z_3 will be image of z_1 w.r.t. x-axis

and z_2 will be on negative x-axis $z_2 = -2$

$z_3 = (1, -\sqrt{3})$



Sol.12 C

$e^{i\theta} . e^{i2\theta} . e^{i3\theta} ... e^{in\theta} = 1$

$e^{\frac{in(n+1)\theta}{2}} = e^{i\theta}$

$e^{\frac{in(n+1)\theta}{2}} = e^{i2m\pi}$

$\frac{n(n+1)\theta}{2} = 2m\pi$

$\theta = \frac{4m\pi}{n(n+1)}$; $m \in z$

Sol.13 C

$x = a + b + c$; $y = a\alpha + b\beta + c$; $z = a\beta + b\alpha + c$

$\alpha = \omega, \beta = \omega^2$

$xyz = (a + b + c)(a\omega + b\omega^2 + c)(a\omega^2 + b\omega + c)$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc$

Sol.14 C

Let $z = x + iy$

$|z - 1|^2 + |z + 1|^2 = 2$

$(x - 1)^2 + y^2 + (x + 1)^2 + y^2 = 2$

$2x^2 + 2y^2 + 1 = 2$

$x^2 + y^2 = 0$ ordered pairs (0, 0)

$\Rightarrow x = 0, y = 0$

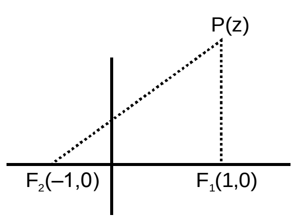
Sol.15 C

$|z - 1| + |z + 1| \le 4$

$PF_1 + PF_2 \le 4$

$F_1F_2 < 4$

So it is a interior part of an ellipse.

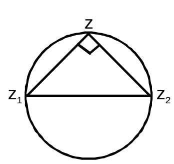


Sol.16 B

$z_1 = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, $z_2 = \frac{-1}{2} - i\frac{\sqrt{3}}{2}$

$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$

$= \left|\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) - \left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right)\right|^2$



$= 3 \Rightarrow \lambda = 3$

Sol.17 D

Le $z = x + iy$

$|z| = Re(z) + z$

$\sqrt{x^2 + y^2} = x + 2$

$x^2 + y^2 = x^2 + 4x + 4$

$y^2 = 4x + 4$

Sol.18 B

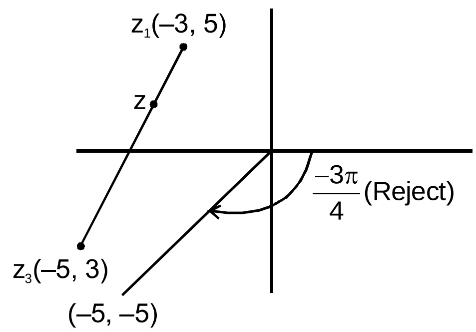
$x^2 + i(a - 1)x + 5 = 0$

coeff. of x should be zero

$a - 1 = 0 \quad \Rightarrow a = 1$

Sol.19 D

$z_1 = -3 + 5i$; $z_2 = -5 - 3i$



Arg. cant be ios 1st and 4th quad.

So (B) & (C) rejected

Sol.20 A

Firest term = $\frac{1}{2}(\sqrt{3} + 1)$

common ratio = $\frac{1}{2}(\sqrt{3} + 1)$

$n \perp h$ term = are^{n-1}

$= \frac{1}{2}(\sqrt{3} + 1)\ \frac{1}{2n}\ (\sqrt{3} + 1)^{n-1}$

$= \frac{1}{2}(\sqrt{3} + i)^n$

$= \frac{2n}{2n}(ei\theta)$

$= e^{i\theta}$

Absolute term = 1

Sol.21 B

$z = x + iy$

$z^{1/3} = a - ib$

$z = (a - ib)^3 = a^3 = a^3 + ib^3 - 3a^abi - 3ab^2$

$x + iy = a^3 - 3ab^2 + i\,(b^3 - 3a^2b)$

$x = a^3 - 3ab^2 \Rightarrow \dfrac{x}{a} = a^2 - 3b^2$

$y = b^3 - 3a^2b \Rightarrow \dfrac{y}{b} = b^2 - 3a^2$

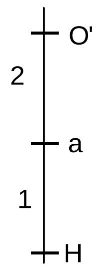
$\dfrac{x}{a} - \dfrac{y}{b} = 4(a^2 - b^2) \qquad k = 4$

Sol.22 D

circumcentre (H) = 0

orthocentre = O^1

centroid = $\dfrac{z_1 + z_2 + z_1}{3}$



a = $\dfrac{2H + a'}{3}$

$\dfrac{z_1 + z_2 + z_3}{3} = \dfrac{2 \times 0 + 0^1}{3} \Rightarrow O^1 = z_1 + z_2 + z_3$

Sol.23 B

$\left(\dfrac{1+i}{1-i}\right)^n = \left(\dfrac{1-i}{1+i}\right)^n$

$\left(\dfrac{1+i}{1-i}\right)^{2n} = 1$

$\left(\dfrac{2i}{2}\right)^{2n} = 1 \qquad \Rightarrow (-i)^{2n} = 1$

n = 2

Sol.24 A

$(a + ib)^5 = \alpha + i\beta$

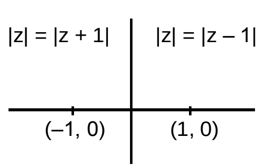
$S = (b + ia)^5$

$= (i)^5 \left(a + \dfrac{b}{i}\right)^5 = i\,(a - ib)^5 = i\,(\alpha - i\beta) = \beta + i\alpha$

Sol.25 D

$|z| = \max\{|z - 1|, |z + 1|\}$

There will not be any possible point whoe distance area equal from (1, 0) & (–1, 0).



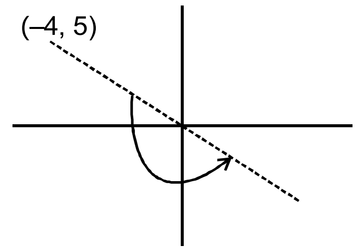
Sol.26 C

$\left.\begin{array}{l} |z_1 - 1| < 1 \Rightarrow |z_1| < 2 \\ |z_2 - 2| < 2 \Rightarrow |z_2| < 4 \\ |z_3 - 3| < 3 \Rightarrow |z_3| < 6 \end{array}\right]$ add $|z_1| + |z_2| + |z_3| < 12$

$|z_1 + z_2 + z_3| < |z_1| + |z_2| + |z_3| < 12$

Sol.27 A

$\dfrac{z_1}{r_1} = \dfrac{z}{r}\, e^{i\pi}$



$\dfrac{z_1}{r_1} = -\dfrac{z}{r}$

$\dfrac{z_1}{1.5r} = -\dfrac{-z}{r}$

$z_1 = -\dfrac{3}{2}\,z = -\dfrac{3}{2}\,(-4 + 5i) = 6 - \dfrac{15}{2}i$

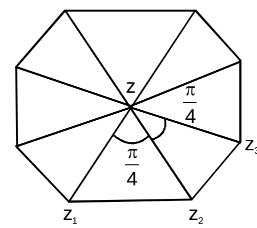
Sol.28 C

By rotation

$z_2 - z = (z_1 - z)\,e^{i\pi/4}$ ...(1)

$z_3 - z = (z_2 - z)\,e^{i\pi/4}$ ...(2)

$z_2 - z_3 = (z_1 - z_2)\,e^{ix/4}$

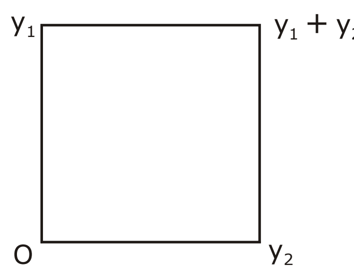


$z_3 = z_2 + (z_2 - z_1)\left(\dfrac{1+i}{\sqrt{2}}\right)$

And similarly $z_3 = z_2 + (z_2 - z_1)\left(\dfrac{1-i}{\sqrt{2}}\right)$

Sol.29 C

$\arg \dfrac{(y_1 + y_2)}{(y_1 - y_2)} = \dfrac{\pi}{2}$



a rhombus not a square

Sol.30 A

$\left[\dfrac{1 + i\tan\theta}{1 - i\tan\alpha}\right]^n - \left(\dfrac{1 + i\tan n\alpha}{1 - i\tan n\alpha}\right)$

$= \left[\dfrac{\cos\alpha + i\sin\alpha}{\cos\alpha - i\sin\alpha}\right]^n - \left(\dfrac{\cos n\alpha + i\sin n\alpha}{\cos n\alpha - i\sin n\alpha}\right)$

$= \left[\dfrac{e^{-i\alpha}}{e^{-i\alpha}}\right]^n - \left[\dfrac{e^{-n\alpha}}{e^{-in\alpha}}\right] = e^{i2n\alpha} - e^{i2n\alpha} = 0$

Sol.31 C

$P = a + b\omega + c\omega^2$; $q = b + c\omega + a\omega^2$; $r = c + a\omega + b\omega^2$

$p + q + r = (a + b\omega + c\omega^2) + (b + c\omega + a\omega^2) + (c + a\omega + b\omega^2)$

$= (a + a\omega^2 + a\omega) + (b\omega + b + b\omega^2) + (c + c\omega + c\omega^2) = 0$

$(p + q + r)^2 = \Sigma p^2 + 2\Sigma Pq = 0 \Rightarrow \Sigma P^2 = -2\Sigma pq$

$P = \omega q$

$P = \omega^2 r$

$SP^2 = P^2 + q^2 + r^2 = P^2 + P^2\omega^2 + P^2\omega = 0$

$\Sigma Pq = Pq + qr + rP = P^2\omega^2 + P^2 + P^2\omega = 0$

$\Sigma P^2 = -2\Sigma Pq$ and $Sp^2 = 2\Sigma pq$

Sol.32 A

$x^2 + x + 1 = 0 - x = \omega$ & $x = \omega^2$

$$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 +$$

$$\left(x^4 + \frac{1}{x^4}\right)^2 + \ldots\ldots.. + \left(x^{27} + \frac{1}{x^{27}}\right)^2$$

$$= 9\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega^2 + \frac{1}{\omega^2}\right)^2 + \left(\omega^2 + \frac{1}{\omega^2}\right)^2 + \ldots\ldots.$$

$$+ \left(\omega^{27} + \frac{1}{\omega^{27}}\right)^2$$

$$= 9\left[\left(\frac{-\omega}{\omega}\right)^2 + \left(\frac{-\omega^2}{\omega^2}\right)^2 + (1+1)^2\right] = 9(1+1+4) = 54$$

Sol.33 A

$\alpha = (1)^{1/5}$ $\quad 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 0$

$z^4 + z^3 + z^2 + z + 1 = 0$ $\quad |\alpha| = 1$

$$2^{|1+\alpha+\alpha^2+\alpha^{-2}-\alpha^{-1}|} = 2^{\left|1+\alpha+\alpha^2+\frac{1}{\alpha^2}-\frac{1}{\alpha}\right|}$$

$$= 2^{\left|\frac{\alpha^2+\alpha^2+\alpha^4+1-\alpha}{\alpha^2}\right|} = 2^{\left|\frac{-2\alpha}{d^2}\right|} = 2^2 = 4$$

Sol.34 C

$z^{10} - z^5 - 992 = 0$

$z^{10} - 32z^5 + 31z^6 - 992 = 0$

$(z^5 - 32)(z^5 + 31) = 0$

$z^5 = 32$

$$z = 2\left(\cos\frac{2m\pi}{5} + i\frac{\sin 2m\pi}{5}\right)$$

$m = 0 \quad z = 2$ ×

$m = 1 \quad z = 2\left(\cos\frac{2\pi}{5} + i\frac{\sin 2\pi}{5}\right)$ ×

$m = 2 \quad z = 2\left(\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\right)$ ✓

$m = 3 \quad z = 2\left(\cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}\right)$ ✓

$m = 4 \quad z = 2\left(\cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5}\right)$ ×

$z^5 = -31$

$$z = (31)^{1/5}\left[\cos\frac{2\pi+\pi}{5} + i\sin\frac{2m\pi+\pi}{5}\right]$$

$m = 0$; $z = (31)^{1/5}\,[e^{ix/5}]$ ×

$m = 1$; $z = (31)^{1/5}\,[e^{-i3\pi/5}]$ ✓

$m = 2$; $z = (3q)^{1/5}\,[e^{ix}]$ ✓

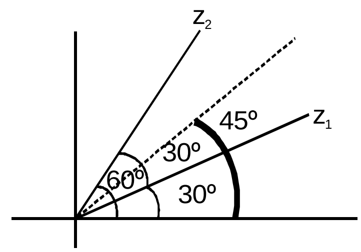
$m = 3$; $z = (31)^{1/5}\,[e^{i7\pi/5}]$ ✓

$m = 4$; $z = (31)^{1/5}\,[3^{i9x/5}]$ ×

Sol.35 B

$z_1 = 3 + \sqrt{3}\,i \quad \arg z_1 = 30°$

$z_2 = 2\sqrt{3} + 6i \quad \arg z_2 = 60°$



argument of angle bisector will be 45º

Sol.36 B

$|z - 3| = 2$ $\quad |z| = 2$

$\Rightarrow (x - 3)^2 + y^2 = 4 \quad x^2 + y^2 = 4$

$(x - 3)^2 + 4 - x^2 = 14$

$-6x + 9 = 0 \Rightarrow x = \frac{3}{2}$ $\quad y = \frac{\sqrt{7}}{2}$, $\quad y = -\frac{\sqrt{7}}{2}$

$z = \frac{1}{2}(3 \pm i\sqrt{7})$

Sol.37 B

$|z - 2| = 3 \quad \Rightarrow (x - 2)^2 + y^2 = 9$

$|z - 2 - 3i| = 4 \quad \Rightarrow (x - 2)^2 + (y - 3)^2 = 16$

Radical axis

$y^2 - (y - 3)^2 = 9 - 16$

$6y - 9 = 9 - 16$

$6y = 9 - 7$

$y = \frac{1}{3}$

Sol.38 A

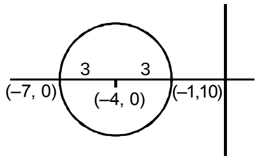
$(1 + i)\, z = (1 - i)\, \bar{z}$

$z = t\,(i - i)$ will satisfy

Sol.39 C

$|z + 1|_{min} = 0$

$|z + 1|_{max} = 3 + 3 = 6$



Sol.40 C

$$S = \sum_{k=1}^{10}\left(\text{sijn}\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$$

$$= \frac{1}{i}\sum_{k=1}^{10}\left(i\sin\frac{2k\pi}{11} - \cos\frac{2k\pi}{11}\right)$$

$$= i\sum_{k=1}^{10}\left(\cos\frac{2k\pi}{11} - i\sin\frac{2k\pi}{11}\right)$$

$$= i\sum_{k=1}^{10} e^{-\frac{i2k\pi}{11}} \qquad \text{Let } a = e^{-i\frac{2\pi}{11}}$$

$$= i\sum_{k=4}^{10}\alpha^k$$

$= i\,(\alpha + \alpha^2 + \alpha^3 + ... + \alpha^{10} + 1 - 1)$

$= i\,(-1) = -i$

Sol.41 B

$(x - 1)^3 = -8$

$(x - 1) = (-2)^{1/3}$

$x - 1 = -2, -2\omega, -2\omega^2$

$x = -1,\ 1 - 2\omega,\ -1 - 2\omega^2$

Sol.42 B

$|z_1 + z_2| = |z_1| + |z_2|$ Let $z_1 = e^{-i\frac{2\pi}{11}}$

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1|\,|z_2|$ $z_2 = r_2\, e^{-i\theta_2}$

$r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2) = r_1^2 + r_2^2 + 2r_1r_2$

$\cos(\theta_1 - \theta_2) = 1$

$\theta_1 - \theta_2 = 0$

$\arg z_1 - \arg z_2 = 0$

Sol.43 B

$$\omega = \frac{z}{z - \frac{1}{3}i}$$

$$|\omega| = \frac{|z|}{|z - \frac{1}{3}i|}$$

$$|z| = \left|z - \frac{1}{3}i\right|$$

$$x^2 + y^2 = x^2 + \left|y - \frac{1}{3}\right|^2$$

$y - \frac{1}{3} = \pm y$ Two st. lines

Sol.44 C

$z = -i\omega$

$\arg = \arg(-i\omega) = \arg(-i) + \arg \omega$

$\theta = \phi - \frac{\pi}{2}$ ...(1)

$\arg z = 0\ \pi$

$\theta + \phi = x$ ...(2)

$\theta = x - \theta - \frac{\pi}{2}$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

Sol.45 B

$|z^2 - 1| = |z^2| + 1$ Let $z = x + iy$

$|x^2 - y^2 - 1 + 2ixy| = \sqrt{(x^2 - y^2) + 4x^2y^2} + 1$

$\sqrt{(x^2 - y^2 - 1)^2 + 4x^2y^2} = \sqrt{(x^2 - y^2 + (4x^2y^2)} + 1$

$(x^2 - y^2 - 1)^2 + 2\,(x^2 - y^2) = (x^2 - y^2)^2 + 4x^2y^2$

$+ 1\ 1 + 2\sqrt{(x^2 - y^2)^2 + 4x^2y^2}$

$(x^2 - y^2) = (x^2 - y^2) + 4x^2y^2$

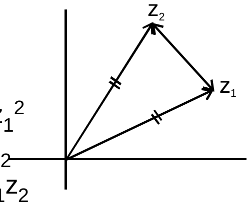
Sol.46 C

$z^2 + az + b = 0 \Big\langle \begin{matrix} z_1 \\ z_2 \end{matrix}$

$z_2 = z_1 e^{i\pi/3}$ ...(1)

$0 - z_1 = (z_2 - z_1)\, e^{i\pi/3}$ ...(2)

$-\dfrac{z_2}{z_1} = \dfrac{z_1}{z_2 - z_1}$



$\Rightarrow \quad -z_2^2 + z_1z_2 = z_1^2$

$z_1^2 + z_2^2 = z_1z_2$

$(z_1 + z_2)^2 = 3z_1z_2$

$z^2 = 3b$

Sol.47 D

$|z\omega| = 1 \qquad \arg z - \arg \omega = \dfrac{\pi}{2}$

$|z|\,|\omega| = 1; \qquad \omega = \dfrac{1}{r}e^{-i\theta}$

$\bar{z}\,\omega = re^{-i\left(\frac{\pi}{2}+\theta\right)} \cdot \dfrac{1}{r}e^{i\theta}$

$= e^{-i\frac{\pi}{2}} = -i$

Sol.48 A

$z_r = e^{-i\frac{\pi}{2^r}}$

$z_1z_2z_3 \;....\; \infty$

$= e^{-i\frac{\pi}{2}} \cdot e^{-i\frac{\pi}{2^2}} \cdot e^{-i\frac{\pi}{2^3}}$

$= e^{-i\pi\left[\frac{1}{2}+\frac{1}{2^2}+...\infty\right]} = e^{-i\pi\left(\frac{1/2}{1-\frac{1}{2}}\right)} = e^{-i\pi} = -1$

Sol.49 B

$\left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1-\cos\dfrac{3\pi}{8}\right)\left(1-\cos\dfrac{\pi}{8}\right)$

$= \left(1-\cos^2\dfrac{\pi}{8}\right)\left(1-\cos^2\dfrac{3\pi}{8}\right)$

Sol.50 C

$z^2 = -1$

$z = -1, -\omega, -\omega^2$

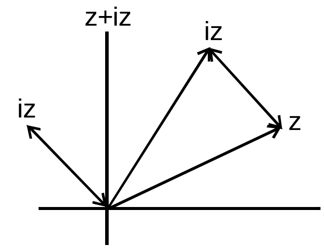
product $= (-1)\,(-\omega)\,(-\omega^2) = \omega^3 - 1$

Sol.51 C

Area $= \dfrac{1}{2}|z|\,|iz|$

$= \dfrac{1}{2}\,|z|\,|z|$

$= \dfrac{1}{2}|z|^2$



Sol.52 C

$2z_2 = z_1 + z_3$

$y_2 = \dfrac{z_1 + z_3}{2}$

lie on line

Sol.53 C

$\sin 3x \sin 3x = \sum\limits_{m=0}^{n} c_1 \cos mx$

$\left(\dfrac{3\sin x - \sin 3x}{4}\right)$ $(\sin 3x)$

$\dfrac{3\sin x \sin 3x}{4} + \dfrac{1}{4}\sin^2 3x$

$\dfrac{3x^2}{8}\sin x \sin 3x + \dfrac{1}{8}\, 2\sin^2 3x$

$\dfrac{3}{8}\,[\cos 4x - \cos 2x] + \dfrac{1}{8}\,[1 - \cos 6x]$

$\dfrac{3}{8}\cos 4x - \dfrac{3}{8}\cos^2 x + \dfrac{1}{8} - \dfrac{1}{8}\cos 6x$

$m = 6$

Sol.54 A

$\dfrac{z_1}{r_1} = \dfrac{z}{r} = e^{i\pi}$

$\dfrac{z_1}{3r} = -\dfrac{-z}{r}$

$z_1 = -3z = -3(4 - 3i)$

$z_1 = -12 + 9i$

Sol.55 B

$|z - z - 3i| + |z + 2 - 6i| = 4$

$z_1 = 2 + 3i$; $z_2 = -2 + 6i$

$z_1z_2 = \sqrt{(2+z)^2 + (3-6)^2} = \sqrt{16+8} = 5$

$z_1z_2 > 2a$ So no locus

Sol.56 B

$|z_1| = 12$ $\quad |z_2 - 3 - 4i| = 5$

$|z_2| \le 10$

$||z_1| - |z_2||$ £ $|z_1+z_2| \le |z_1| + |z_2|$

$|z_1 - z_2 \ge |z_1| - |z_2||$

$\ge |12 - 10|$

$|z_1 - z_2| \ge 2$

Sol.57 A

$|z_1| = |z_2| = |z_3| = 1$

$\bar{z}_1 = \dfrac{1}{z_1}$; $\bar{z}_2 = \dfrac{1}{z_2}$; $\bar{z}_3 = \dfrac{1}{z_3}$

$\left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3}\right| = 1$

$|\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 1$

$|\overline{z_1 + z_2 + z_3}| = 1$

$|z_1 + z_2 + z_3| = 1$

Sol.58 C

$1 + z + z^2 + z^3 + ... + z^{n-1} = \dfrac{z^n - 1}{z - 1}$

$= \dfrac{(z-1)(z-\alpha)(z-\alpha^2)...(z-\alpha^{n-1})}{(z-1)}$

$\dfrac{z^n - 1}{z - 1} = (z - \alpha)(z - \alpha^2) ... (z - \alpha^{n-1})$

put $z = 3$ $\dfrac{3^n - 1}{2} = (3 - \alpha)(3 - \alpha^2) ... (3 - \alpha^{n-1})$

Sol.59 A

$(1 + i)x^2 - (7 + 3i)x + (6 + 8i) = 0$

$x_1x_2 = \dfrac{6+8i}{1+i}$

$x_2(4 - 3i) = 2i\ \dfrac{(4-8i)}{1+i}$

$x_2 = \dfrac{2i}{1+i} \times \dfrac{1-i}{1-i} = \dfrac{2i(1-\alpha)}{2}$

$x_2 = i + 1$

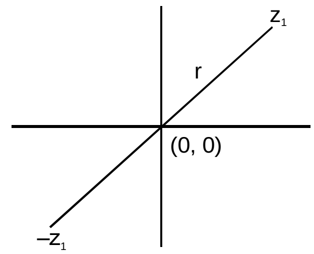
Sol.60 A

$|z - 0| = |z_1|$

$\left|\dfrac{z}{z_1}\right| = 1$

$\dfrac{z}{z_1} \cdot \dfrac{\bar{z}}{\bar{z}_1} = 1$

$z\bar{z} = z_1\bar{z}_1$



Sol.61 D

$z = x + iy$

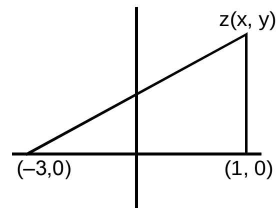
Given that

$\dfrac{y-0}{x+3} = \dfrac{y-0}{x-1}$

$\Rightarrow \dfrac{1}{x+3} - \dfrac{1}{x-1} = 0$

$x - 1 - x - 3 = 0$

non-senge



Sol.62 A

$\log_{1/2}|z - 2| > \log_{1/2}|z|$

$|z - 2| < |z|$

$(x - 2)^2 + y^2 < x^2 + y^2$

$-4x + 4 < 0$

$x > 1 \quad \Rightarrow Re(z) > 1$

Sol.63 D

$z^3 + \bar{z} = 0$

$r^3 = r$

$r = 0, \quad r = 1$

$z = 0, \quad z\bar{z} = 1$

$z^3 + \dfrac{1}{z} = a$

$z^4 = -1$ No. of solutions = 4

Total no. of solutions = 4 + 1 = 5

Sol.64 D

$iz^3 + z^2 - z + i = 0$

$z = i$ wil satisfy above equation $\quad \Rightarrow |z| = 1$

Sol.65 C

$|z - a^2| + |z - 2a| = 3$

$3 > z_1z_2$

$|2a - a^2| < 3$

$-3 < 2a - a^2 < 3$

$2a - a^2 > -3$	$2a - a^2 < 3$
$a^2 - 2a - 3 < 0$	$a^2 - 2a + 3 > 0$
$-1 < a < 3$	$a \in R$
as $a > 0$	
$0 < a < 3$	$a \in (0, 3)$

Sol.66 A

$$\sum_{k=0}^{n-1} | z_1+\omega^2 z_2 |^2$$

$$= \sum_{k=0}^{n-1} (z_1+\omega^k z_2)(\bar{z}_1+\bar{\omega}^k \bar{z}_2)$$

$$= \sum_{k=0}^{n-1} (| z_1 |^2 + \bar{\omega}^k z_1\bar{z}_2 + \omega^2 \bar{z}_1 z_2 + | z_2 |^2)$$

$$= n\,(|z_1|^2 + |z_2|^2) + z_1\,\bar{z}_2\ \underbrace{\bar{\omega}^0+\bar{\omega}^1+\bar{\omega}^2+\ldots+\bar{\omega}^{n-1}}_{=0 \text{ as sum of } n^{th} \text{ roots}}$$

$$+$$

$$z_1\,\bar{z}_2\ \underbrace{\omega^0+\omega^1+\omega^2+\ldots+\omega^{n-1}}_{=0 \text{ as sum of } n^{th} \text{ roots}}$$

$$= n\,(|z_1|^2 + |z_2|^2)$$

Sol.67 D

$|z_1| = 2, |z_2| = 3, |z_3| = 4$

$|z_1|^2 = 4, \qquad z_2\,\bar{z}_2 = 9,\ z_3\,\bar{z}_3 = 16$

$z_1\,\bar{z}_1 = 4$

$|8z_2z_3 + 27z_3z_1 + 64\ z_1z_2|$

$$= |z_1z_2z_3|\ \left|\frac{8}{z_1}+\frac{27}{z_2}+\frac{64}{z_3}\right|$$

$= 2 \times 3 \times 4\ |2\,\bar{z}_1 + 3\,\bar{z}_2 + 4\,\bar{z}_3\,|$

$= 24\ |\,\overline{2z_1+3z_2+4z_3}\ = 24 \times 4 = 96$

Sol.68 D

$4\,(z_1 - z_2) = 3\,(z_2 - z_2)$

$$\frac{z_1-z_2}{3} = \frac{z_2-z_2}{4}\ e^{-i0}$$

$$\frac{z_1-z_2}{z_2-z_3} = \text{Purely real}$$

Sol.69 C

$Ax + By + C = 0$

$\bar{a}\,z + a\,\bar{z} + 2C = 0$

$$x = \frac{z+\bar{z}}{2}\ ; \qquad y = \frac{z-\bar{z}}{2i}$$

$$A\left(\frac{z+\bar{z}}{2}\right) + B\left(\frac{z-\bar{z}}{2i}\right) + C = 0$$

$A\,(z + \bar{z}) - Bi(z - \bar{z}) + 2C = 0$

$(A - Bi)\,z + (A + Bi)\,\bar{z} + 2C = 0$

By Comparing

$a = A + iB$

Sol.70 A

$|z| = 2$

$$E = |z_1 + z_2 + \ldots + z_n| - 4\left|\frac{1}{z_1}+\frac{1}{z_2}+\ldots+\frac{1}{z_n}\right|$$

$$E \le (|z_1| + |z_2| + \ldots + |z_n|) - 4\left(\frac{1}{|z_1|}+\frac{1}{|z_2|}+\ldots+\frac{1}{|z+1|}\right)$$

$$\le 2b - 4\left(\frac{n}{2}\right)$$

$E \le 0$

Sol.71 D

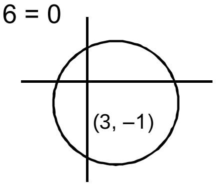
$z\,\bar{z} - (3 + i)\,z - (3 - i)\ \bar{z} - 6 = 0$

Put $z = x + iy$

$x^2 + y^2 - 6x + 2y - 6 = 0$

$C(3, -1)\ r = 4$

Infinite points



Sol.72 D

$1 + x^2 = \sqrt{3}\,x$

$x = -\,i\omega,\ i\omega^2$

$$\sum_{n=1}^{24}\left(x^4 - \frac{1}{x^n}\right)$$

$$x - \frac{1}{x} + \left(x^2 - \frac{1}{x^2}\right) + \ldots\ \left(x^3 - \frac{1}{x^3}\right) + \ldots + \left(x^{24} - \frac{1}{x^{27}}\right)$$

$$= 3\left[\left(-i\omega - \frac{1}{i\omega^2}\right) + \left(x^3 - \frac{1}{x^3}\right) + \ldots + \left(x^{24} - \frac{1}{x^{24}}\right)\right]$$

$= 0$

Sol.73 A

$$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$$

$R_1 \to R_1 + R_3$

$$= \begin{vmatrix} 1-i & \omega+\omega^2 & \omega^2-1 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 1-i & -1 & \omega^2-1 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} = 0$$